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Three-dimensional turbulent boundary layers: a report on EUROMECH 33

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The 33rd EUROMECH Colloquium on three-dimensional turbulent boundary layers was held in Berlin during 25–27 September 1972. Forty-two participants from six countries were present, and Professor R. Wille and the author organized the Colloquium. The papers presented could be clearly divided into two groups, one dealing with prediction methods and the other describing experimental results. References quoted give the titles of the papers presented at the meeting and refer to further work discussed at the Colloquium; there will be no other publication of the proceedings.

1. Prediction methods for three-dimensional turbulent boundary layers

As with two-dimensional boundary layers, calculation methods for three-dimensional turbulent boundary layers can be conveniently divided into two groups which are referred to as 'differential' and 'integral'. In three-dimensional flows all methods involve the solution of partial differential equations by finite-difference methods, but in the case of integral methods the number of dimensions is reduced from three to two by first integrating the equations across the complete boundary layer before seeking a solution. This reduction in the number of dimensions generally results in a significant reduction in the computational time required by integral methods, but this is achieved at a cost, at least potentially, in accuracy and in limitations on the complexity of flows that can be calculated. Within each type of calculation method many particular procedures can be written which may differ both in the particular empirical information used to obtain closure of the equations and in the generality of the co-ordinate system used. Many calculation methods are written in streamline co-ordinates, in which the co-ordinates on the wall are the projections of the external streamlines and their orthogonal trajectories, but greater facility in calculating a range of different flow configurations can be acquired if more general co-ordinates are used, particularly if they are not necessarily orthogonal.

1.1. *Differential methods*

P. Wesseling & J. P. F. Lindhout (1971) have published their calculation method for three-dimensional incompressible turbulent boundary layers using Bradshaw's (1971) vector equation for the turbulent shear stress. A Courant–Isaacson–Rees difference scheme was used for solving the differential equations.

Comparisons of the theoretical results with two new sets of measurements were presented and the importance of the influence of the approximation functions for the boundary conditions (e.g. the pressure distribution at the outer edge of the boundary layer) on the calculation was discussed. Good agreement was obtained with A. J. Vermeulen's experiment I, where the streamwise pressure gradient was small. Greater discrepancies between theory and experiment became apparent, however, if strong adverse pressure gradients were present both in Vermeulen's second test case and in the NLR swept wing experiment.

E. Krause, W. Kordulla & E. H. Hirschel described a calculation method which was originally developed for compressible laminar boundary layers (Krause, Hirschel & Bothmann 1969), and which was extended to turbulent flows using R. Michel's eddy viscosity model. Preliminary investigations had dealt with computational details such as the relative errors that arise from different step sizes and the number of iterations used. Test calculations have shown that laminar, transitional and turbulent boundary layers can be calculated by this method.

T. K. Fannelop reported on a quasi two-dimensional calculation method along streamline co-ordinates for three-dimensional turbulent boundary layers, claiming the advantage of shorter computing time. The three-dimensional equations are written in an equivalent two-dimensional form where all terms containing the lateral variable appear as forcing functions. These terms are evaluated by solving the boundary-layer equations along two neighbouring streamlines. If a first approximation to these solutions is obtained initially (using the small cross-flow assumption) the desired lateral derivatives can be evaluated, and the solution can be improved by iteration. The Van Driest and the Clauser eddy viscosity models were used for the inner and outer layer respectively. The equations were transformed to a simliarity form and solved by an implicit finite-difference method. Though the method can easily be extended to various boundary conditions (e.g. suction, etc.) further comparisons with experiments are needed.

B. Roux & P. Bontoux reported on a calculation method for the boundary layer in the plane of symmetry on a cone at incidence in supersonic flow using locally similar solutions which were obtained by the application of the usual Lees–Dorodnitsyn transformation and a variable y/δ respectively, where y is the distance from the wall and δ is the boundary-layer thickness. By this method, details of which have not yet been published, it was possible to predict the effect of incidence on the skin friction, the heat-transfer rate at the wall and the deflexion of the limiting streamlines close to the plane of symmetry. The case of the equivalent laminar boundary layer was described by Roux (1972).

J. J. Delgado Domingos discussed the effect upon the stability of numerical calculation procedures of the cell Reynolds number and showed that its influence was dependent upon the turbulence model used as well as other parameters. The conclusions reached apply to both explicit and implicit finite-difference schemes and suggestions for improving the precision of calculation schemes were made (see also Domingos & Filipe 1972).

1.2. *Integral methods*

Four integral methods were presented, all of which are based upon the solution of the mean momentum equations integrated across the boundary layer. All the methods make use of the entrainment equation and differ essentially in the form of the velocity profile families that are assumed and in the generality of the co-ordinate systems used.

J. P. Bernard indicated two ways of solving three-dimensional turbulent boundary-layer problems. First he introduced the Crocco transformation into the partial differential equations for momentum in the streamwise and orthogonal directions. The cross-flow profile was given by Eichelbrenner's (1965) velocity profile, which is capable of describing cross-over profiles. After integration, an integral equation dependent on three parameters representing the pressure gradient and the Reynolds number was obtained. The second calculation method was more conventional (Galerkin–Kantorovitch) and used an entrainment function, a Coles type skin-friction law and a two-parameter family of cross-flow profiles. A comparison of the theoretical skin-friction and shape parameter values with experiments on a body of revolution at an angle of incidence of 3·5° showed good agreement.

J. Cousteix & C. Quémard used a local similarity hypothesis to obtain the streamwise velocity and enthalpy profile families. Then, having solved the equations in the streamwise direction under the assumption of small cross-flow, a solution for the equation in the transverse direction can be found if a similarity assumption of the type $\overline{w}/\overline{u}_\delta = f(y/\delta)$ is made, where $\overline{w}$ is the cross-flow velocity and $\overline{u}_\delta$ is the external flow velocity. The turbulent shear stress and heat flux were calculated by means of an eddy viscosity model based on an extension of their earlier (1972) mixing-length model to three-dimensional boundary layers. The resulting velocity profiles are functions of Reynolds number, Mach number, heat flux at the wall and two pressure-gradient parameters (Michel, Quémard & Cousteix 1971). This method allows the calculation of cross-over velocity profiles, and comparison with experiment has shown that profiles with large cross-flows are also well represented. C. Quémard & J. Cousteix have incorporated the velocity profile families developed in the previous paper into an integral calculation method for incompressible boundary layers both with and without rotation of the wall. The cross-flow velocity profile family contains $\Omega\delta/u_\delta$ as a parameter, where Ω is the angular velocity. Comparisons of the theoretical results showed good agreement with the measured boundary layers on swept wings, in front of a blunt body and for a swirling boundary layer in a vaneless diffuser.

P. D. Smith described his calculation method, which used the boundary-layer momentum integral and entrainment equations in a non-orthogonal curvilinear co-ordinate system. The system of hyperbolic equations (Myring 1970) was solved by an explicit finite-difference method in which the forward step size was controlled by the requirement that the dowstream point remained within the domain of influence of the upstream point and its immediate neighbours. Both the Mager and the Johnston representations of the cross-flow velocity profile could be used. Comparisons were presented of predictions of this calculation method with the experimental results of A. J. Vermeulen, Johnston (1960), and L. F. East. Significant discrepancies were apparent in the prediction of the streamwise momentum thickness in A. J. Vermeulen's curved duct flow but the remaining comparisons showed good agreement. A feature of the program is the general form of the co-ordinate system used, which enables a wide range of flow configurations to be computed without modification to the program.

W. L. Lindsay discussed an integral method which had been developed by the Turbomachinery Group at Cambridge. Head's entrainment equation and Coles' velocity profile for the streamwise flow were used as auxiliary equations and the cross-flow profile was determined from the Johnston profile with an improved relationship for the determination of the matching point between the inner and outer part of the boundary layer. Comparisons were made with experiments on a stationary hub in a flow with high swirl and an adverse pressure gradient. Under these rather severe flow conditions the agreement with experiment was poor.

1.3. *Discussion*

At the end of this series of talks the relative merits of integral and differential methods for three-dimensional turbulent boundary layers were discussed. There were good reasons for the use of both: the more rapid integral methods are well suited for design purposes while differential methods are applicable to more complex flows and to the study of the turbulence mechanism. Strong emphasis was given to the desirability of close co-operation between those developing calculation methods and those designing experiments such as had taken place at N.L.R. Such co-operation helps to ensure that the quantities required as input to calculation procedures are measured in sufficient detail and are presented in a form convenient for numerical calculation. At this stage of the discussion a meeting similar to the Stanford Conference was suggested so that the accuracy of the available prediction methods for three-dimensional turbulent boundary layers could be compared. J. C. Rotta & R. Michel declared their willingness to follow up this proposal and to survey and catalogue the available experimental data. Suggestions for further experiments will be made if the survey should show apparent gaps in the experiments available so far.

2. Measurements in three-dimensional turbulent boundary layers

2.1. *Measurements on infinite swept wings and on a slender wing*

Four speakers described experiments in turbulent boundary layers which were designed to approximate to infinite swept wing conditions. The best results appear to have been obtained when curved end walls were installed, the curvature of which was approximately that of the streamlines of the potential flow for the infinite swept wing. The range of the sweep angle was between 20° and 45°. In three test cases the swept wind conditions were simulated by the flow along a swept flat plate with an opposite wall shaped so as to generate the adverse pressure gradient. A test section of this type excludes *a priori* any investigation of the influence of curvature on the turbulence in the boundary layer which may occur in many practical cases such as on aircraft wings. Static pressure, flow direction, and mean velocity profiles were measured by all experimenters. All the wind tunnel models had been designed to provide test cases for a comparison with calculation methods.

G. Redeker discussed mean velocity profiles in boundary layers with four different pressure gradients and with cross-flow angles β_w at the wall, up to 40°. Since β_w close to separation had been well predicted by the calculation method of Cumpsty & Head (1967) in this case, it was considered that this method might be well suited for application in a more extensive calculation procedure for the prediction of buffet onset in transonic boundary layers on swept wings.

D. W. Etheridge had performed eight runs with various pressure gradients, two of them leading to separation of the boundary layer. The experiments he described had mild pressure gradients and were thus well suited for a comparison with theory. Several different methods were used to determine the wall shear stress but no conclusive answer about a best method could be given. There was satisfactory agreement between the measurements and the predictions of both Bradshaw's (1971) calculation method and of Smith's (1968) integral entrainment method.

M. C. P. Firmin reported on measurements near the trailing edge of a swept wing in which great care was taken to ensure that the measurements were not influenced by the support mechanism. Owing to the high Reynolds number of the flow (chord Reynolds number up to 10^7) the interference from an externally mounted traverse gear was likely to be unacceptable and so the probes were mounted from within the wing section and it was necessary to calibrate the yaw meters for local Mach number and yaw. The corrections made to the measurements were discussed in detail.

B. Van den Berg & A. Elsenaar gave a presentation of their very elaborate experiments (Reynolds number $2{\cdot}42 \times 10^6\,\mathrm{m}^{-1}$) in which special attention had been given to obtaining as close an approximation as possible to infinite swept wing conditions. As in A. J. Vermeulen's experiment the velocity profiles in the boundary layer were measured with a rotatable hot wire protruding through the plate surface and the skin friction was determined by means of Preston tubes and Stanton tubes. The measurements of the flow direction could be reproduced to within 0·2° except of angles close to the wall. It was found that the pressure distribution over the test surface could be expressed satisfactorily by a double polynomial of seventh and fourth degree, the coefficients of which were given in a table. Good agreement was found between skin friction measured by means of a wall Pitot tube and wall shear stress determined from a Clauser plot where the resultant velocity was used. Skin friction was found to decrease in the chordwise direction, reaching a minimum in the region of separation of the three-dimensional boundary layer, where the wall streamlines were parallel to the leading edge, and increasing again after separation. The momentum balance applied to the boundary-layer measurements confirmed the accuracy of the measured results and suggested that the measurements are reliable except very close to separation.

L. F. East discussed measurements of skin friction (Preston tube and razor blade technique) and mean velocity profiles in the boundary layer on the lifting surface of a half model slender wing at incidence with a root chord of 7.3 m. The boundary layer under the leading-edge vortex had limiting streamlines with angles up to 8° except close to separation. From the momentum equations the shear stress distribution through the boundary layer had been estimated across the span of the model using the detailed measurements of velocity profiles and skin friction which had been made at three chordwise stations. The shear stress distribution deduced was shown to be consistent with an eddy viscosity model in which the eddy viscosity normal to the local mean velocity was approximately 40 % of the eddy viscosity parallel to the mean velocity. Consequently considerable differences occurred between the direction of shear stress and the mean velocity gradient through the boundary layer. The eddy viscosity parallel to the mean velocity could be represented by a mixing-length formulation similar to that in two-dimensional flow.

2.2. *Measurements in different flow configurations*

A. J. Vermeulen gave an account of detailed measurements made in the boundary layer of a 60° curved duct with straight entry and exit sections. The first set of data was measured in a flow with effectively zero longitudinal pressure gradient and high rates of cross-flow in the curved section, while the second set of data was measured in a flow with a strong adverse pressure gradient leading to separation and displaying increased rates of cross-flows coupled with divergence. Mean velocity profiles were taken with a hot wire and a flattened three-tube yaw meter, both protruding through the wall. Skin friction was determined by a surface fence calibrated against a Preston tube. This measurement was confirmed by calculations using the streamwise and transverse momentum equations. The measured mean velocity profiles were used to calculate the shear stress profiles and the 'effective' viscosity through integration of the equations of motion.

L. Larsson had intended to report on boundary-layer measurements on a two metre reflex model of a cargo ship at a Reynolds number of 5×10^6 in air. Owing to damage to the wind tunnel, however, only the measuring techniques could be described.

H. Fernholz & J.-D. Vagt discussed preliminary measurements of skin friction and mean and fluctuating velocities along the circumference and at several stations in the streamwise direction of the boundary layer in a symmetric annulus fitted with an asymmetric end plate. The three-dimensional boundary layer generated in this way is subject to strong adverse pressure gradients and curvature effects. A brief survey was given on surface fences as a means of measuring the magnitude and the direction of the wall shear stress.

The next two investigations dealt with the separation line of the three-dimensional flow pattern in front of a circular cylinder, of variable height-to-diameter ratio between 0·2 and 8, aligned perpendicular to a fixed ground plate. The oncoming boundary-layer thickness on the ground plate was approximately constant and smaller than the cylinder diameter. R. Falco & R. Kinns had measured the distance between the upstream separation line and the stagnation point on the cylinder and that of the primary horseshoe vortex and the stagnation point. They found the values for both distances to be independent of height-to-diameter ratio providing that this ratio exceeds unity, whereas the position of the 'wall wake separation line' showed a distinct maximum. The latter result was confirmed by F. Etzold and E. Mercker, who, however, found a distinct maximum for the distance of the forward separation line plotted against the height-to-diameter ratio. This difference in the behaviour of the forward separation line can be explained by the state of the boundary layer, which was turbulent in the first case and laminar in Etzold's experiment.

D. Hughes & J. H. Horlock gave a description of turbulent boundary layers on the stationary or rotating hub of an annular duct with either zero or adverse pressure gradient in an axial or swirling outer flow. The independent variables were the swirl angle outside the boundary-layer flow and the rotational speed of the hub. Velocity profiles and shear stress profiles through the boundary layer were measured for several combinations of these parameters.

2.3. *Discussion*

The discussion group on measuring techniques in three-dimensional turbulent boundary layers dealt with four topics: the measurement of flow direction, wall shear stress, static pressure profiles and shear stress profiles. Two- and three-tube yaw meters or hot wires have been used to measure the direction of the flow. In the region very close to the wall the hot-wire probe seemed to have an advantage over the yaw meter though problems concerning corrections due to heat conduction are not yet solved. Correction problems occur also with yaw meters in the vicinity of the wall. In this context M. C. P. Firmin pointed out errors due to misalignment of a three-tube yaw meter on a test wall.

The well-known techniques for the measurement of skin friction in two-dimensional boundary layers have been extended to the three-dimensional case (Preston tube, Stanton tube, razor blade and surface fence) with the additional condition that the height of the measuring device should be as small as possible since large variations of the flow angle can exist close to the wall. This height restriction causes problems in measuring very small pressure differences and in avoiding contamination due to dust particles in the pressure tappings. Since discrepancies between the static pressure measured by wall tappings and the static pressure measured by static probes within the boundary layer and in the external flow could not be explained satisfactorily in some test cases, it was agreed to compare the static pressure probes used in different laboratories in several test flows.

The most difficult problem has been the measurement of the shear stress profile. For this purpose the N.L.R. group had extended A. J. Vermeulen's rotatable hot-wire probe to an array consisting of two X-wire probes protruding through the wall, which had the slight disadvantage, however, that the position of the X-wire was not in the axis of rotation. L. F. East suggested the use of either triple hot-wire probes or split film probes, but neither of these types of probes has yet been properly developed for this purpose. The cross-beam laser technique was mentioned as a means of measuring the shear stress distribution close to the wall, but here again considerable development of the technique is required. Unfortunately none of those present had had any practical experience with this measuring technique in turbulent air flows.

3. Miscellaneous topics

J. L. Peube & J. L. Tuhault presented several samples of very clear flow pictures of wall streamlines in curved ducts, on rotating disks, turbine blades, and on inclined circular cylinders, which had been obtained by an electro-chemical reaction on the copper wall of the respective flow geometry. Flow visualization of this type has been achieved because the rate of mass transfer is slightly different in the wake of a very small protrusion on the wall. As a consequence wall streamlines were etched into the wall if only tiny natural irregularities were present. The underlying physical phenomena were discussed briefly.

A.K. Rastogi described an investigation into the effectiveness of three-dimensional film cooling slots. The experimental part covered the influence of the velocity ratio (coolant velocity to free-stream velocity), of the density ratio and of geometrical parameters such as the open-area ratio. The cooling slots were made up of discrete holes which discharged the coolant parallel to the surface to be cooled. In a second paper a comparison was given between the results of a calculation method developed by Spalding & Patankar and Rastogi's experiments. The numerical procedure was based on the solution of a finite-difference representation of the time-averaged boundary-layer equations which took account of the cross-stream recirculation in the flow. A generalized form of Prandtl's mixing-length formula for three-dimensional boundary layers was used. The agreement between theoretical results and experiments was remarkably good, particularly in view of the very few mesh points that had been used in the direction normal to the wall in this rather complicated flow configuration.

4. Conclusions

No time was left for a panel discussion at the end of the Colloquium owing to extended meetings of the discussion groups on 'numerical methods' and 'experimental techniques'. A comparison with the surveys of Horlock, Norbury & Cooke (1965) and Cooke (1965) shows, however, that many questions raised at EUROMECH 2 are still unanswered. More is known about the flow in corners and wing–body junctions, at least on the experimental side, but no computation method presented at the Colloquium dealt with this type of flow where the y and z derivatives are an order of magnitude greater than the derivatives in the main-flow direction. As far as true three-dimensional boundary layers are concerned, very little is known about separation both experimentally and theoretically though the development of computational techniques has made considerable progress.

Future experiments in three-dimensional boundary layers should be designed carefully, so that they give useful test data for numerical methods or provide new and reliable information about the turbulence structure of the boundary-layer flow. In the first case shear-driven (rather than pressure-driven) test boundary layers (e.g. Moore & Richardson 1957; Bradshaw & Terrell 1969) with accurately known external flow conditions are necessary. In the second case a thorough knowledge of hot-wire or laser techniques and data processing is essential. This seems the only way to improve existing turbulence models and to establish a general velocity profile family necessary for integral methods.

The author has drawn freely on papers and abstracts provided by the participants of the Colloquium. Dr L. F. East of the Royal Aircraft Establishment made valuable suggestions and amendments during the preparation of this paper and Prof. E. Krause provided his notes about the discussion on 'numerical methods'. Their assistance is gratefully acknowledged.

REFERENCES

(An asterisk by a name indicates a lecture given at the Colloquium)

BERNARD, J. P.* Methode de résolution des equations de couches limites tridimensionelles turbulentes incompressibles et applications.

BRADSHAW, P. 1971 Calculation of three-dimensional turbulent boundary layers. *J. Fluid Mech.* **46**, 417–445.

BRADSHAW, P. & TERRELL, M. G. 1969 The response of a turbulent boundary layer on an infinite swept wing to the sudden removal of pressure gradient. *N.P.L. Aero. Rep.* no. 1305.

COOKE, J. C. 1965 Three-dimensional boundary layers. *R.A.E. Tech. Memo. Aero.* no. 915.

COUSTEIX, J. & QUÉMARD, C. 1972 Velocity profiles and wall shear stress of three-dimensional turbulent boundary layers. *ONERA, T.P.* no. 1134. Edition provisoire.

COUSTEIX, J., QUÉMARD, C. & MICHEL, R. 1972 Application d'un schéma amélioré de longueur de mélange à l'étude des couches limites turbulentes tridimensionelles. *AGARD Current Paper*, no. 93.

CUMPSTY, N. A. & HEAD, M. R. 1967 The calculation of three-dimensional turbulent boundary layers. *Aeron. Quart.* **18**, 55–84, 150–164.

DOMINGOS, J. J. D.* Development of a numerical solution procedure for three-dimensional turbulent boundary layers.

DOMINGOS, J. J. D. & FILIPE, J. M. C. 1972 A comparative study of difference schemes for the solution of $\partial u_i/\partial t + u_j\, \partial u_i/\partial x_j = \nu\nabla^2 u_i + \partial F/\partial x_i,$. *3rd Int. Conf. on Numerical Methods in Fluid Mech.*

EAST, L. F.* Measurements of the turbulent boundary layer on a slender wing.

EICHELBRENNER, E. 1965 La couche limite tridimensionelle en régime turbulent d'un fluide compressible: cas de la paroi athermane. *AGARDograph*, no. 97.

ETHERIDGE, D. W.* Some results of recent research at Queen Mary College.

ETHERIDGE, D. W. 1971 An experimental investigation into three-dimensional turbulent boundary layers in incompressible flow. Submitted for Ph.D. degree at the University of London.

ETZOLD, F. & MERCKER, E.* Short communication on boundary-layer separation caused by circular cylinders of variable height-to-diameter ratios protruding from a flat plate.

FALCO, R. & KINNS, R.* The sensitivity of the skin friction field and separation lines of a three-dimensional turbulent boundary layer to changes in Reynolds number and pressure distribution.

FANNELOP, T. K.* A simple finite-difference procedure for solving the three-dimensional laminar and turbulent boundary-layer equations.

FERNHOLZ, H. & VAGT, J.-D.* Measurement in a three-dimensional incompressible turbulent boundary layer.

FIRMIN, M. C. P.* Measurements of the mean flow in three-dimensional turbulent boundary layers near the rear of a swept wing at subsonic speeds and moderate Reynolds numbers.

HORLOCK, J. H., LINDSAY, W. L. & PERKINS, H.* An integral method for the calculation of three-dimensional boundary layers and comparison with experiment.

HORLOCK, J. H., NORBURY, J. F. & COOKE, J. C. 1967 Three-dimensional boundary layers: a report on Euromech 2. *J. Fluid Mech.* **27**, 369–380.

HUGHES, D. & HORLOCK, J. H.* Swirling boundary-layer flow formed on stationary and rotating cylinders.

JOHNSTON, J. P. 1960 On the three-dimensional turbulent boundary layer generated by secondary flow. *J. Basic Engng, Trans. A.S.M.E.* D **82**, 233.

KRAUSE, E., HIRSCHEL, E. H. & BOTHMANN, TH. 1969 Differenzenformeln zur Berechnung dreidimensionaler Grenzschichten. *DLR FB* no. 69–66.

KRAUSE, E., KORDULLA, W. & HIRSCHEL, E. H.* Finite-difference solutions for three-dimensional turbulent boundary layers.

LARSSON, L.* Measurements in the three-dimensional boundary layer of a cargo ship.

LE BOT, Y.* Rôle de la couche limite turbulente dans les compresseurs axiaux.

MICHEL, R., QUÉMARD, C. & COUSTEIX, J. 1971 Solution de similitude pour les couches limites turbulentes avec gradient de pression en fluide compressible. *La Recherche Aérospatiale ONERA*, no. 1971–6, pp. 323–333.

MOORE, R. W. & RICHARDSON, D. L. 1957 Skewed boundary-layer flows near the end walls of a compressor cascade. *Trans. A.S.M.E.*, **79**, 1789–1800.

MYRING, D. F. 1970 An integral prediction method for three-dimensional turbulent boundary layers in incompressible flow. *R.A.E. Tech. Rep.* no. 70/47.

PATANKAR, S. V., RASTOGI, A. K. & WHITELAW, J. H.* The effectiveness of three-dimensional film cooling slots: predictions.

PEUBE, J. L. & TUHAULT, J. L.* A new electrochemical visualization process of wall streamlines in three-dimensional turbulent boundary layers.

QUÉMARD, C. & COUSTEIX, J.* 1972 Calculation methods for three-dimensional turbulent boundary layers with and without rotation of the walls. *ONERA Tech. Paper*, no. 1135, Edition Provisoire.

RASTOGI, A. K. & WHITELAW, J. H.* The effectiveness of three-dimensional film cooling slots: measurements.

REDEKER, G.* 1970 Measurements in the three-dimensional turbulent boundary layer on an infinite swept wing. *DFVLR Ber.* no. 0627. Braunschweig.

ROUX, B. 1972 Supersonic laminar boundary layer near the plane of symmetry of a cone at incidence. *J. Fluid Mech.* **51**, 1–14.

ROUX, B. & BONTOUX, P.* Couche limite turbulent dans le plan de symetrie d'un cone de révolution dans un courant supersonique.

SMITH, P. D.* An integral prediction method for three-dimensional compressible turbulent boundary layers.

SMITH, P. D. 1968 Calculation method for three-dimensional turbulent boundary layers. *Aero. Res. Counc. R. & M.* no. 3523.

VAN DEN BERG, B. & ELSENAAR, A.* 1972 Measurements in a three-dimensional turbulent boundary layer under infinite swept wind conditions. *Draft Version NLR Tech. Rep.* 72092 U.

VERMEULEN, A. J.* 1971 Measurements of three-dimensional turbulent boundary layers. Ph.D. thesis, University of Cambridge.

WESSELING, P. & LINDHOUT, J. P. F.* Three-dimensional incompressible turbulent boundary layers: comparison between calculations and experiments.

WESSELING, P. & LINDHOUT, J. P. F.* 1971 A calculation method for three-dimensional incompressible turbulent boundary layers. *AGARD Current Paper*, CPP-93-71.